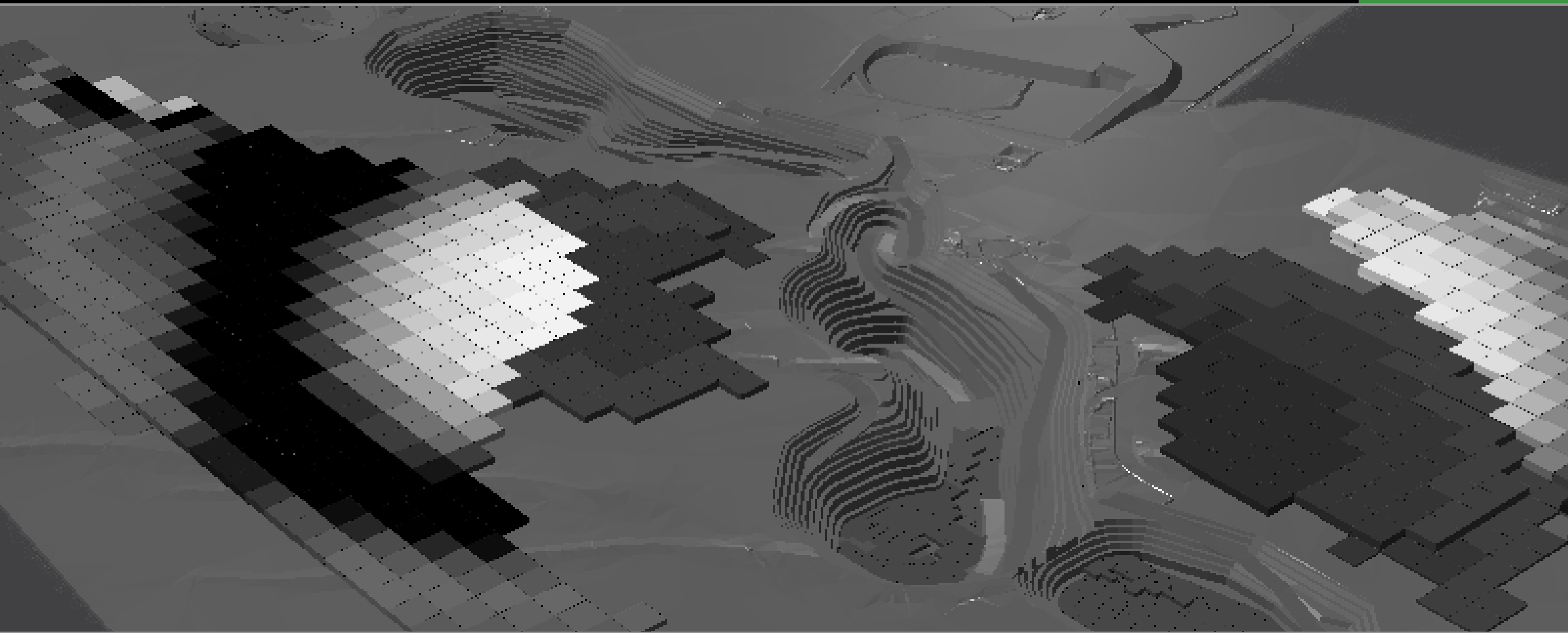
MAPTEK™ Evolution



Applying Modern Heuristics to Maximising NPV through Cut-off Grade Optimisation

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ABSTRACT

In focusing on open pit mine scheduling, one of the biggest areas where a mining engineer can add value is in the area of cut-off grade optimisation. It is not atypical to add up to an extra 20% in value by bringing forward higher grade ore and deferring costs into the future.

There are many techniques available to maximise net present value when applied to open pit mine scheduling. That in itself raises the question of why so many of these techniques exist. The short answer is that none of these techniques are robust. Every time the problem changes, the algorithms also have to change.

Instead of trying to make a problem fit the constraints of an existing technique, there is much to be said for usefully hybridizing different approaches. This paper describes an innovative approach where evolutionary algorithms are combined with both local search and linear programming. The goal is maximising net present value (NPV) through optimisation of cut-off grade policy and extraction sequence for an open pit mine. The cut-off/cut-over grade policy produced caters for multiple processing streams and stockpiles.

Finally we demonstrate the quality of the algorithm by comparing some results with a well-known mathematical solution.

INTRODUCTION

There exists a substantial arsenal of problem-solving techniques or algorithms that have been developed to address a variety of problems. Unfortunately as everyone knows, it's almost always the case that the real world presents us with circumstances that are different to varying degrees than are required by these techniques. In the rush to present problem solving techniques people often tend to use an "off the shelf" classical problem solving technique. That often results in forcing a particular real world problem to fit the constraints of a particular technique. "Better solutions to real-world problems can often be obtained by usefully hybridizing different approaches" (Michalewicz and Fogel, 2004). Recognising the complexity of real-world problems is prerequisite to their effective solution.

People often blindly chase after the "optimal" without thinking about the implications thereof and what "optimal" actually implies. It should be remembered that every time a problem is solved in reality we only find a solution to a model of the problem. All models are a simplification of the real world, otherwise they would be as unwieldy and complicated as the real setting itself. The solution is only a solution in terms of the model and every model is associated with a set of assumptions and leaves something out. What this implies is that when it comes to solving complex real world problems with classical methods we mostly end up finding a precise solution to an approximate model (Michalewicz and Fogel, 2004). It should be clear that most real world problems do not yield to classical methods; if they did, they would not be problems anymore. Finally, it pays to remember that the very nature of optimality (in the face of multiple constraints) implies that the solution will often be brittle. More often than not it will reside at the border between feasible and infeasible. A slight variation from the optimal can result in a dramatic change in the objective or worse, lead to infeasibility as illustrated by figure 1.

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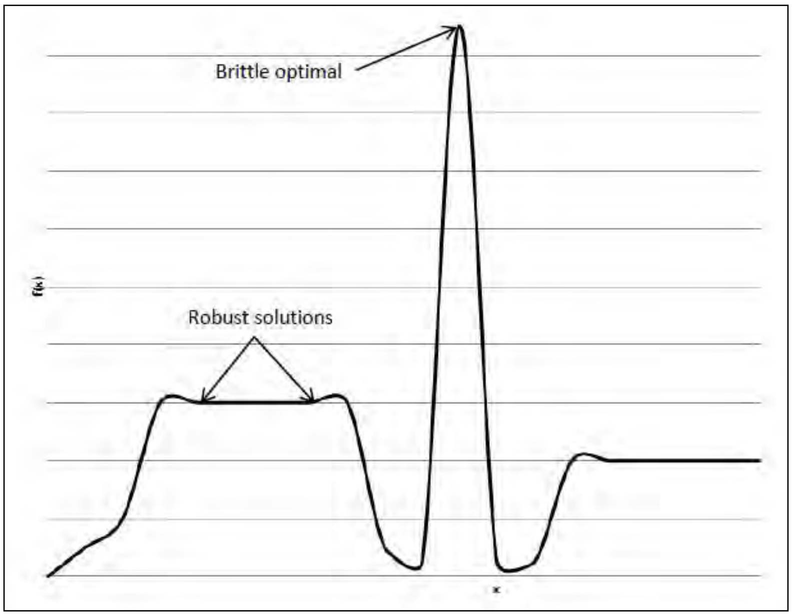

Figure 1 – Brittle optimal.

In light of the above it seems prudent to not blindly cling to any particular approach or method, but rather to embrace all techniques when attempting to solve complex real world problems. Good examples of this can be found in articles (Lamghari and Dimitrakopoulos, 2012 and Myburgh and Deb, 2010) where metaheuristics were applied to the open pit mine production scheduling problem.

In solving the problem of maximising NPV by optimising cut-off grade and extraction sequence we have hybridized modern heuristics with classical optimisation techniques. The objective function here is neither continuous nor differentiable (Ataei and Osanloo, 2004) making it rather difficult to solve by most classical or gradient techniques in particular. To maximise a project's NPV Lane (1988) showed that dynamic rather than constant breakeven cut-off grades should be used. Cut-off grade optimisation has long been recognised for its potential to improve a project's value. Cut-off grade optimisation can be performed considering different objectives, but maximising net present value is the most applicable.

Lane developed an algorithm that maximises a project's NPV by taking the time value of money into account under three constraints (mine, mill and market). The strategy was based on raising the cut-off grade above the breakeven cut-off grade. Lane showed that to maximise the NPV one has to include the fixed costs of not receiving future cash flows earlier due to the cut-off grade decisions that one takes today. These concepts are further explained elsewhere (Lane 1988, Rendu 2008, Dagdelen 1992, 1993, 1995 and Whittle and Wharton 1995).

Unfortunately Lane's algorithm cannot be applied directly if one wants to optimise cut-off grade in the face of multiple processing streams, stockpiles or multi-element commodities (Dagdelen and Kawahata, 2007). Lane's algorithm furthermore only optimises cut-off grades for a predetermined extraction sequence. If faced with any of the aforementioned complexities, Lane's algorithm has to be extended (Asad 2002, 2005, 2007 and Gholamnejad 2009). A hybridization of evolutionary and classical algorithms is used by Evorelution Strategy to overcome the limitations stated above. Evorelution was acquired by Maptek during 2014 to be marketed globally as part of their geological modelling and mine planning solution.

Evolutionary algorithms have come a long way since their inception in the 1960s. Their roots can be traced back all the way to Holland (1962, 1975). Although he did not develop his genetic algorithm (GA) framework for solving optimisation problems per se, one of his PhD students, Kenneth Dejong (1975), demonstrated through a simulation study that Holland's GA can be used as a competitive optimisation algorithm to classical numerical unconstrained optimisation algorithms. However, Holland's interest was to develop a nature-inspired methodology for creating self-adaptive systems. Interestingly, it is this self-adaptiveness of a GA that contributes greatly to its ability to successfully address complex real-world problems. Its agility allows it to be easily hybridized with other optimisation methods. Here the hybridized or memetic algorithm employs three levels of optimisation: A "master" evolutionary algorithm which manages two other "slave" or lower level optimisation algorithms. The master algorithm manages variation of cut-off grades as well as permutations of the extraction sequence. In addition, it manages a linear programming algorithm which is responsible for determining the optimal flow of material through multiple processing streams as well as management of the stockpile policy. Finally, every so often, a third local search technique is called upon to provide a push or boost to the best or fittest schedule so far.

MODEL

The problem under discussion is the simultaneous optimisation of extraction sequence and cut-off grade for a single element in the face of multiple processing streams. Prerequisites for the development of a cut-off grade optimisation model include:

- Development of an ultimate pit limit or pushback or some portion inside the ultimate pit limit that can be mined, processed and refined in a number of years.
- Mining, processing and refining capacities.
- Operating costs and metal price.
- Proper stage design is optional but preferred. The impact of proper stage designs on cut-off grade optimisation cannot be stressed enough.
- The ore reserves inside the pit limit or pushback in terms of mineral grade and tonnage distribution. A grade tonnage distribution is calculated for each phase-bench combination.

The objective function of Strategy's cut-off grade optimisation model is maximisation of NPV in the presence of capacity constraints (mine, mill and market and stockpile), multiple processing streams and extraction sequence. It can be represented mathematically as follows:

$$Max\ NPV = \sum_{n=1}^{N} \frac{P_n}{(1+d)^n}$$

Where:

d = discount rate

$$P = profit\ (\$) = \left(P_{process_1} + P_{process_2} + \cdots + P_{process_n}\right) - fT$$

And

$$P_{process_n} = (s-r)\bar{g}_n y_n Q_{c_n} - mQ_{m_n} - cQ_{c_n}$$

s = price ($/unit of product)

r = selling cost ($/ unit of product)

$\bar{g}_n$ = average grade of material presented at process n

y_n = recovery at process n

m = mining cost ($/ton)

c = processing cost ($/ton)

f = fixed cost ($/year)

T = length of period considered

Q_{c_n} = quantity of ore presented at process n

Q_{m_n} = quantity of material presented at process n

ENGINE

The engine consists of an effective hybridization of two evolutionary and one classical optimisation algorithm (Figure 2). These include:

- The core or master evolutionary algorithm
- Local search evolutionary algorithm
- Linear programming algorithm

The main responsibilities for each algorithm include:

- Master
    - o Exploring process cut-off grade search space.
    - o Exploring stockpile cut-off grade search space.
    - o Exploring extraction sequence search space.
    - o Manage Local Search Evolutionary algorithm.
    - o Manage Linear Programming Algorithm.
- Local Search
    - o Exploring the immediate neighbourhood of process and stockpile cut-off space for a given extraction sequence, in other words the local search keeps the extraction sequence static.
- Linear Programming algorithm
    - o Optimises the flow of material through available processes.
    - o Responsible for optimal reclaim strategy from stockpiles.

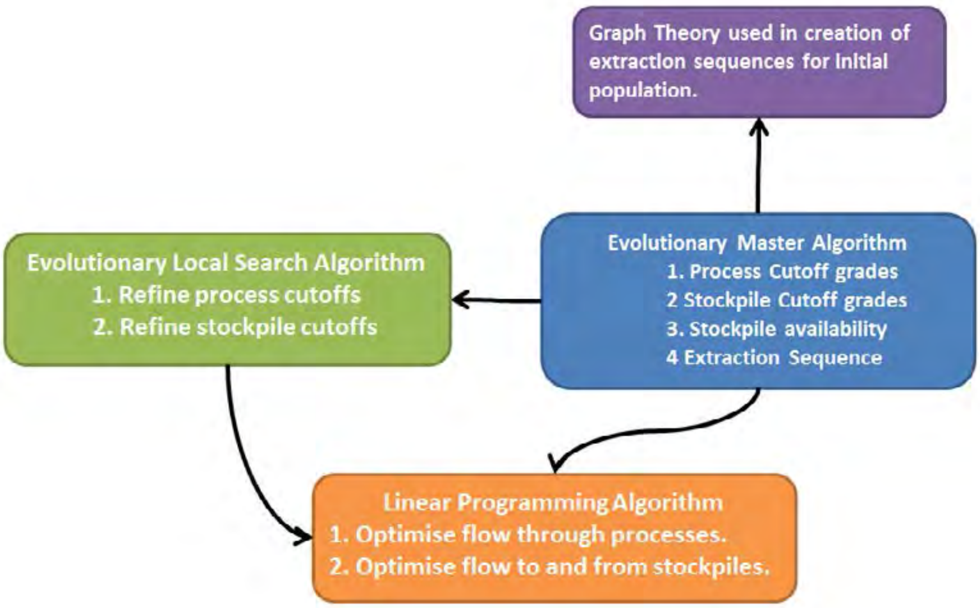

Figure 2 – Strategy engine.

A description of the main steps includes:

1. Creation of an initial population as follows:
    a. For each individual solution a geometrically correct extraction sequence is produced using a combination of graph theory techniques, user supplied constraints, if any, and user supplied economic inputs.
    b. A multi process cut-off grade profile is then generated for each individual. One can think of the profile as strings or threads through cut-off grade space (Figure 3).
    c. Similarly a multi stockpile cut-off grade profile is generated for each individual.
    d. Finally a stockpile availability profile is created for each individual.

2. The fitness of each individual is calculated and the population is ranked based on fitness. The fitness here is NPV. During fitness calculations both the master and local search algorithms call on the linear programming algorithm to use the supplied cut-off grade threads, stockpile availability and extraction sequence to optimise the flow of material through the available processing streams. The linear programming algorithm is also responsible for determining the optimal reclaim strategy.
3. The master then iterates through successive generations by generating an offspring population where each child competes with the parents for the privilege to progress to the next generation.
4. Every so often, the master algorithm calls on the secondary local search algorithm to boost the best individual found so far. The local search does this by manipulating the threads through cut-off grade space whilst keeping the extraction sequence static. The improved individual is then sent back to the master where it replaces or upgrades its old self (analogue to exploring the local neighbourhood).
5. Steps 2 to 4 are repeated until no improvement in NPV is registered, in other words when the population loses diversity and converges on a single high quality NPV.

For an in depth discussion on how evolutionary algorithms work we refer the interested reader to Affenzeller et al (2009), Deb (2001), Deb and Agrawal (1995), Gen and Cheng (1997), Goldberg et al (1989, 1990), Deb and Deb (2012), Deb and Jain (2011) and Goldberg (1989). Here we will briefly elaborate on representation (chromosome) and operators responsible for exploration of the cut-off grade search space.

EXPLORATION OF CUT-OFF GRADE SPACE

1. Representation

As mentioned one can think of a cut-off grade policy or profile as threads running through cut-off grade space. Figure 3 shows an example of what an individual in the initial population's cut-off grade profile might look like (representing five different processing routes).

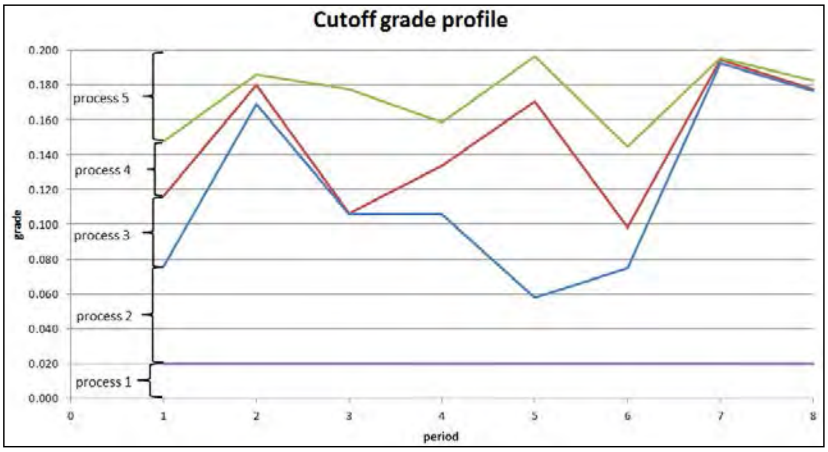

Figure 3 – Example of cut-off grade profile through cut-off grade space.

Stockpile cut-off grades are treated in a similar way. The availability of stockpiles is represented as binary vectors (Figure 4):

- o 0 => stockpile not available
- o 1 => stockpile is available

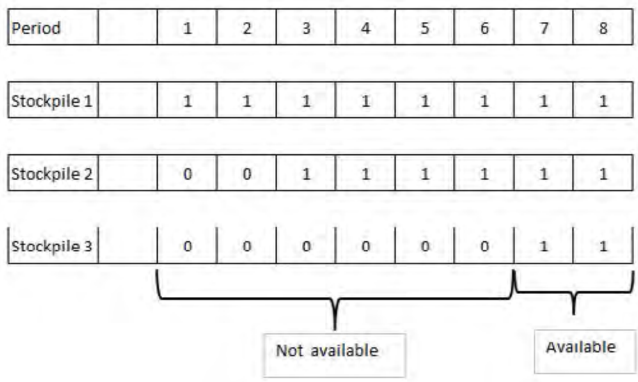

Figure 4 - Stockpile availability.

The extraction sequence is represented by a phase/bench time vector where the position in the vector represents the time when the phase/bench combination was mined. Each vector therefore represents a unique temporal path through the mine model under consideration (Figure 5).

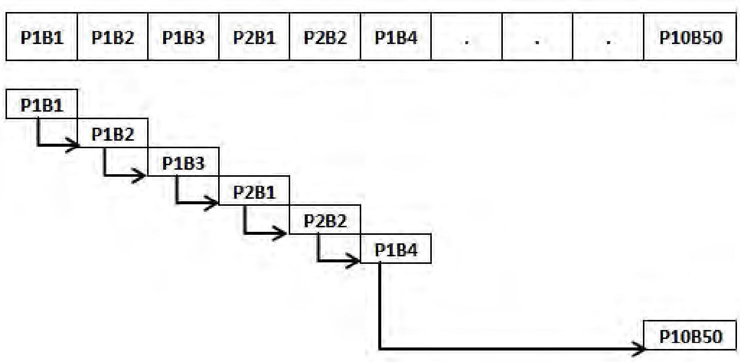

Figure 5 - Time vector representing a temporal path through model.

2. Strategy Genetic Operators

Crossover Operators

In evolutionary algorithms the crossover operator is usually the main tool used for exploring the search space (Goldberg, 1989). Generally speaking, a well-designed evolutionary algorithm functions as a building-block assembling machine and the intention is to combine those parts of the parent solutions that define them as high quality solutions. Ideally, exactly those parts or building blocks of the chromosomes of above average parents should be transferred to the next generation that makes these individuals above average. According to the building block theory one can expect an evolutionary or genetic algorithm in particular to systematically collect the essential pieces of genetic information which are initially spread over the chromosomes of the initial population (Affenzeller, Winkler, Wagner and Beham, 2009). Therefore a well-designed crossover operator should support the potential development of higher-order building blocks (longer allele sequences). To this end it is critical that the problem representation (chromosome) allows a crossover to fulfill this important requirement.

Here the purpose of the crossover operator is two-fold. In terms of cut-off grade threads it has to thoroughly search the random initial cut-off grade threads to create good threads. Thereafter good portions of these threads have to be combined to form better threads. Traditionally binary strings were used to discretise continuous search spaces (Yun, Lian, Lu, Chen, Guo, 2003, Ataei and Osanloo, 2004, Cetin and Dowd, 2001). However, the coding of real-valued variables in finite-length strings causes a number of difficulties such as (Deb and Agrawal, 1994):

- Inability to achieve arbitrary precision
- Fixed mapping of problem variables
- Inherent Hamming cliff associated with binary coding
- Processing of Holland's schemata in continuous search space.

In light of the above, Strategy uses a heavily modified self-adaptive, variable-wise simulated binary crossover (SBX) (Deb and Jain, 2011) operator to recombine cut-off grade threads from multiple parents. The operator is parent centric (in contrast to mean centric) meaning that offspring are created around one of the participating parents. In addition it has the ergodic property such that any real value in the search space can be created from any two parent values, but with differing probabilities. Figure 6 shows the distribution of 10,000 offspring that was created around parents located at x=100 and x=300. A probability distribution is used to control the spread of offspring around the parents. From the figures it is clear that the probability of creating offspring points closer to the parents is larger than that of offspring far away from the parents. The distribution and hence the spread of offspring around parents is dynamically controlled and is varied over the life of the optimisation.

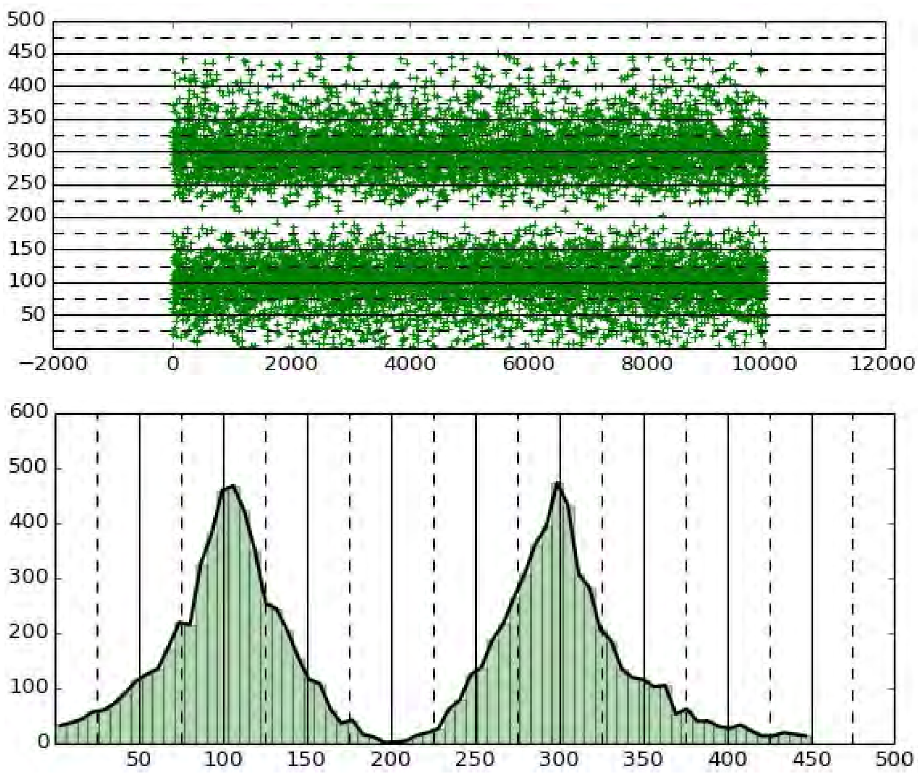

Figure 6 - Parent centric SBX crossover

To crossover the extraction sequences between two parents a different custom operator was developed. This operator combines extraction sequences from 2 parents in such a way that all geometric and user supplied stage release criteria is honored. In fact all of Strategy's crossover operators are developed in such a way that they always produce feasible offspring. This is critical for two reasons:

- Although the application of a repair procedure afterwards to address infeasibility is common practice it has the consequence that alleles or values of genes of the resulting offspring are not present in the parents which directly contradicts the aforementioned building block aspect.
- It eliminates unnecessary computational time being spent on repairing infeasible solutions.

Mutation Operator

It is well known that the mutation operator is mostly used to maintain diversity in the population and is very important in evolutionary algorithms (Deb and Deb, 2012 and Goldberg, 1989). Arguably its most important role is to prevent premature convergence by randomly sampling new points in the search space. In contrast to a recombination operator, a mutation operator operates on only one member of the population at a time and modifies it independent to the rest of the population. Again, early researchers used binary coded genetic algorithms and subsequently mutation was usually implemented as a bit-wise operator attempting to mutate every bit with a probability $p_m = 1/L$ (where L is the total number of bits used to present all variables). It was realised that a major shortcoming of the aforementioned when it comes to real valued problems was the artificial discreteness associated with the coding mechanism as well as the bias of mutated solutions to certain parts of the search space (Deb and Deb, 2012).

In Strategy a heavily modified variable wise polynomial mutation operator is used. In this operator a statistical distribution is used to perturb a solution in a parent's vicinity. To reduce computational complexity the Mutation Clock scheme is used to decide which variables are to be mutated (Goldberg, 1989). The spread of the mutated offspring around the parent is dynamically varied over the life of the optimisation. At the start there is a larger probability to create the offspring further from the parent than towards the end of the optimisation (Figure 7).

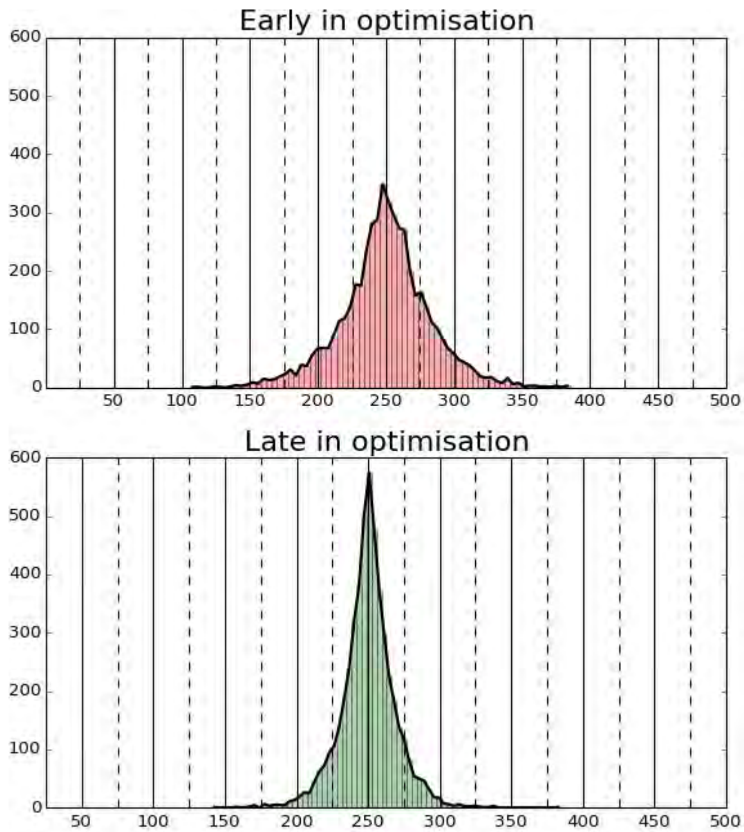

Figure 7 - Strategy polynomial mutation operator.

Hybridized approach vs OptiPit®

To demonstrate the quality of the results that can be obtained by using Strategy we compared it to a well-known commercial optimisation package OptiPit®. In Dagdelen and Kawahata, 2007, they applied OptiPit® to a gold mine example (McLaughlin gold deposit, Northern California, USA) given as a case study in Dagdelen (1992). We will do the same here. As before, the example does not include any complexities related to phase and bench sequencing, but it does serve to highlight the search capability of Strategy in a complex real life operation with multiple processes.

Table 1 shows the grade tonnage distribution of the gold deposit case study and Table 2 the economic model and operational parameters.

Table 1 – Grade tonnage distribution of a gold case study after Dagdelen and Kawahata, 2007.

Interval	From (oz/ton)	To (oz/ton)	MidPoint (oz/ton)	Ktons (oz/ton)
1	0	70	0.01	70000
2	0.02	7.257	0.023	7257
3	0.025	6.319	0.028	6319
4	0.03	5.591	0.033	5591
5	0.035	4.598	0.038	4598
6	0.04	4.277	0.043	4277
7	0.045	3.465	0.048	3465
8	0.05	2.428	0.053	2428
9	0.055	2.307	0.058	2307
10	0.06	1.747	0.063	1747
11	0.065	1.64	0.068	1640
12	0.07	1.485	0.073	1485
13	0.075	1.227	0.078	1227
14	0.08	1.799	0.085	1799
15	0.09	1.799	0.095	1799
16	0.1	0.371	0.105	371
17	0.11	0.371	0.115	371
18	0.12	0.371	0.125	371
19	0.13	0.371	0.135	371
20	0.14	0.371	0.145	371
21	0.15	0.371	0.155	371
22	0.16	0.371	0.165	371
23	0.17	0.371	0.175	371
24	0.18	0.371	0.185	371
25	0.19	0.371	0.195	371
26	0.2	5.864	0.279	5864

Table 2 - Economic and operational parameters (after Dagdelen and Kawahata, 2007).

Price	600	$/oz
Sales Cost	5	$/oz
Mining Cost	1.2	$/ton
Recovery	90	%
Processing Cost	19	$/ton
Fixed Cost	8.35	M
Mining Capacity	Unlimited	
Processing Capacity	1.05	M
Discount Rate	15	%

The following discussion will focus on highlighting the quality of the result that can be obtained by embracing modern heuristics and by creatively combining it with classical optimisation techniques. The quality of the result is shown by comparing it to results obtained by OptiPit® (Dagdelen and Kawahata, 2007).

We start by showing the operating cash flows and schedule obtained based on breakeven cut-off grades for a single Autoclave process (Table 3).

Table 3 - Yearly ore and waste schedules using the breakeven cut-off grades (COG) (after Dagdelen and Kawahata, 2007).

Year	Mining (Mtons)	COG (oz/ton)	Avg Grade (oz/ton)	Processing (Mtons)	Refining (koz)	Profits ($M)
1	3.60	0.04	0.10	1.05	96.30	33.0
2	3.60	0.04	0.10	1.05	96.30	33.0
3	3.60	0.04	0.10	1.05	96.30	33.0
4	3.60	0.04	0.10	1.05	96.30	33.0
5	3.60	0.04	0.10	1.05	96.30	33.0
6	3.60	0.04	0.10	1.05	96.30	33.0
7	3.60	0.04	0.10	1.05	96.30	33.0
8	3.60	0.04	0.10	1.05	96.30	33.0
9	3.60	0.04	0.10	1.05	96.30	33.0
10	3.60	0.04	0.10	1.05	96.30	33.0
11 to 34	3.60	0.04	0.10	1.05	96.30	33.0
35	3.40	0.04	0.10	1.00	96.30	33.0
Total	125.80			36.70	3365.9	1154.2 (NPV @ 15%) 218.5

Tables 4 and 5 shows yearly schedules for optimum cut-off grades for a single process by both OptiPit® (Dagdelen and Kawahata 2007) and Strategy respectively. Figure 8 shows a comparison between the two cut-off grade policies. The two schedules produced are virtually identical with a difference of 0.1% between NPVs. This is quite remarkable considering that the underlying methodologies and algorithms could not be more different. It illustrates the point that what we optimise is really only the model that is used to simplify reality, nothing more.

Table 4 - OptiPit® optimum cut-off grade (COG) schedule for single process.

Year	Mining (Mtons)	COG (oz/ton)	Avg Grade (oz/ton)	Processing (Mtons)	Refining (koz)	Profits ($M)
1	18.40	0.160	0.261	1.05	246.60	96.5
2	16.90	0.150	0.253	1.05	239.10	93.9
3	16.10	0.140	0.248	1.05	234.40	92.0
4	14.70	0.120	0.238	1.05	224.90	87.9
5	14.10	0.110	0.233	1.05	220.20	85.8
6	13.60	0.100	0.228	1.05	215.50	83.6
7	11.00	0.094	0.202	1.05	190.90	72.1
8	8.20	0.070	0.171	1.05	161.60	58.0
9	6.80	0.060	0.152	1.05	143.60	49.3
10	5.50	0.050	0.133	1.05	125.70	39.7
Total	125.80			10.50	2002.50	758.8 (NPV @ 15%) 414.4

Table 5 – Strategy optimum cut-off grade (COG) schedule for single process.

Year	Mining (Mtons)	COG (oz/ton)	Avg Grade (oz/ton)	Processing (Mtons)	Refining (koz)	Profits ($M)
1	17.55	0.156	0.257	1.05	242.7	95.0
2	16.66	0.145	0.251	1.05	237.6	93.1
3	16.03	0.137	0.247	1.05	233.7	91.5
4	15.11	0.123	0.240	1.05	227.2	88.8
5	14.37	0.111	0.234	1.05	221.6	86.3
6	14.06	0.105	0.232	1.05	219.0	85.1
7	10.83	0.086	0.200	1.05	189.0	71.2
8	8.55	0.072	0.174	1.05	164.7	59.4
9	6.87	0.061	0.153	1.05	144.6	49.5
10	5.49	0.050	0.134	1.05	126.2	40.2
Total	125.80			10.50	2006.22	760.1 (NPV @ 15%) 413.8

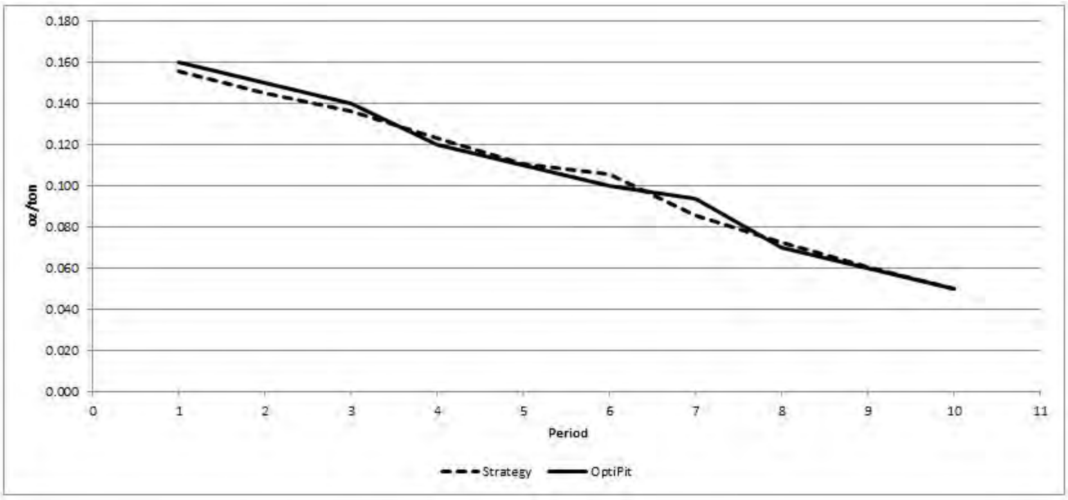

Figure 8 – Comparison of optimum cut-off grade policy for OptiPit® and Strategy.

Let us take this one step further and optimise cut-off grades in the face of multiple processing streams and unlimited mining capacity. Figure 9 shows the additional processes and associated capacity constraints. Table 6 shows the associated processing costs and recoveries.

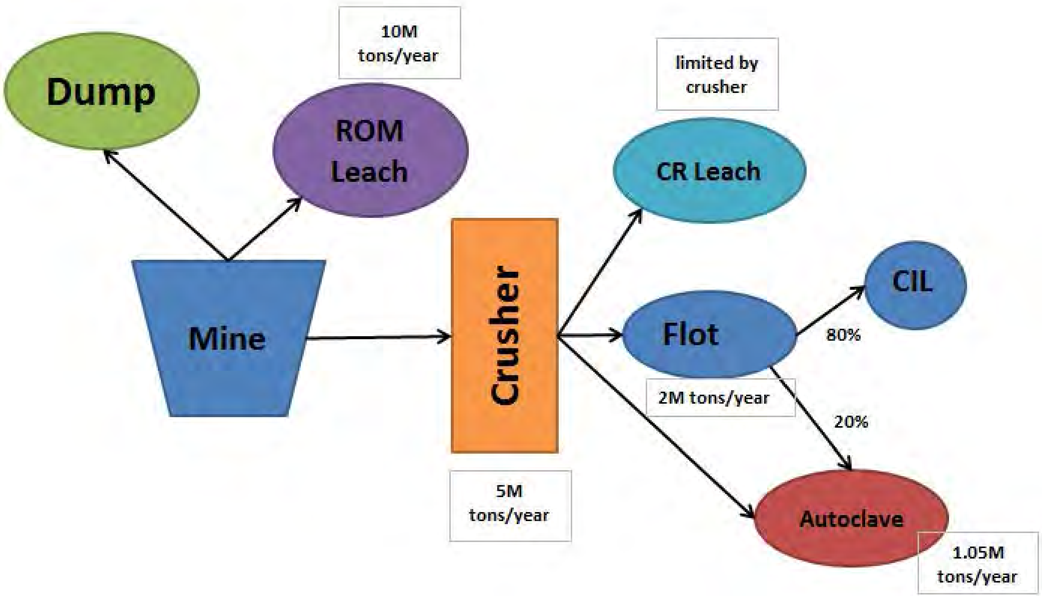

Figure 9 – Diagram showing different material flow routes (after Dagdelen and Kawahata, 2007).

Table 6 – Processing costs and recoveries for different processes (after Dagdelen and Kawahata, 2007).

Processes	Processing Cost ($/ton)	Recovery
ROM LCH	0.90	0.075
CRUSHER LCH	3.00	0.25
FLOT TO AUTO	10.25	0.66
DIRECT AUTO	19.00	0.90

Table 7 shows the yearly schedules using breakeven cut-off grades for multiple processes. The interested reader is referred to Dagdelen and Kawahata, 2007 for details regarding the calculations to determine breakeven cut-off grades between different processes.

Table 7 – Yearly schedules for multiple processes using breakeven cut-off grades (COG) (after Dagdelen and Kawahata, 2007).

Year	Mining	CR Lch			Flot to Autoclave			Direct Autoclave			Refining	Profits
	(Mtons)	COG	Avg grade	Processing	COG	Avg grade	Processing	COG	Avg grade	Processing	(koz)	($M)
		(oz/ton)	(oz/ton)	(Mtons)	(oz/ton)	(oz/ton)	(Mtons)	(oz/ton)	(oz/ton)	(Mtons)		
1	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
2	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
3	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
4	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
5	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
6	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
7	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
8	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
9	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
10	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
11 to 22	5.54	0.02	0.025	0.6	0.03	0.042	1.00	0.06	0.153	0.85	148.4	53.4
23	3.71	0.02	0.025	0.4	0.03	0.042	0.67	0.06	0.153	0.57	99.6	35.8
Total	125.5			13.6			22.7			19.3	3,364.4	1,211.4
												NPV @ 15%
												$341.20

Tables 8 and 9 show the yearly optimised schedules and optimum cut-off grades obtained by OptiPit® and Strategy respectively.

Table 8 – Yearly optimised schedules and optimum cut-off grades obtained by OptiPit® (COG) (Dagdelen and Kawahata, 2007).

Year	Mining	ROM Lch			CR Lch			Flot to Autoclave			Direct Autoclave			Refining	Profits
	(Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	($M)	
1	33.91	0.02	0.036	10	0.062	0.078	2.35	0.099	0.209	2.00	0.20	0.279	0.65	511.9	215
2	25.25	0.02	0.033	6.17	0.049	0.062	2.35	0.08	0.162	2.00	0.20	0.279	0.65	428.7	179.4
3	19.3	0.02	0.029	3.54	0.039	0.05	2.35	0.066	0.125	2.00	0.20	0.279	0.65	365.3	151.1
4	4.96	0.02	0.025	1.62	0.03	0.041	2.55	0.057	0.094	1.75	0.20	0.279	0.7	313.5	128.2
5	13.46	0.02	0.023	0.95	0.026	0.037	2.43	0.05	0.084	1.91	0.19	0.274	0.67	294.7	118.8
6	10.57				0.02	0.029	2.02	0.04	0.066	2.00	0.15	0.254	0.65	250.4	97.4
7	8.05				0.02	0.027	1.23	0.035	0.054	1.60	0.09	0.208	0.73	202	76.5
Total	125.5			22.3			15.3			13.3			4.7	2366.5	966.4 NPV @ 15% $625.2

Table 9 – Yearly optimised schedules and optimum cut-off grades (COG) obtained by Strategy.

Year	Mining	ROM Lch			CR Lch			Flot to Autoclave			Direct Autoclave			Refining	Profits
	(Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ton)	Avg grade (oz/ton)	Process-ing (Mtons)	(koz)	($M)
1	29.8	0.02	0.035	8.15	0.057	0.071	2.35	0.091	0.188	2.00	0.199	0.279	0.65	473.90	199.0
2	26.5	0.02	0.034	6.69	0.051	0.065	2.37	0.084	0.170	1.97	0.199	0.279	0.66	441.62	185.2
3	20.1	0.02	0.030	3.86	0.041	0.052	2.38	0.069	0.131	1.96	0.200	0.279	0.66	374.77	155.7
4	14.0	0.02	0.024	1.18	0.028	0.038	2.38	0.052	0.088	1.97	0.197	0.277	0.66	302.96	122.6
5	13.9	0.02	0.024	1.12	0.028	0.038	2.38	0.051	0.087	1.97	0.198	0.278	0.66	301.24	121.8
6	11.5	0.02	0.021	0.07	0.021	0.032	2.47	0.045	0.072	1.85	0.158	0.258	0.68	265.74	105.0
7	9.8	0.02	0.020	0.01	0.020	0.020	1.84	0.040	0.063	1.77	0.117	0.238	0.70	231.57	91.8
Total	125.5			21.08			16.18			13.48			4.65	2391.8	981.1 NPV @ 15% $625.9

Figure 10 shows the comparison between results from OptiPit® and Strategy. Once again, considering the widely different engines, it is quite remarkable how similar the results are. The NPV and resulting schedules are for all practical purposes the same (0.1% difference in NPV).

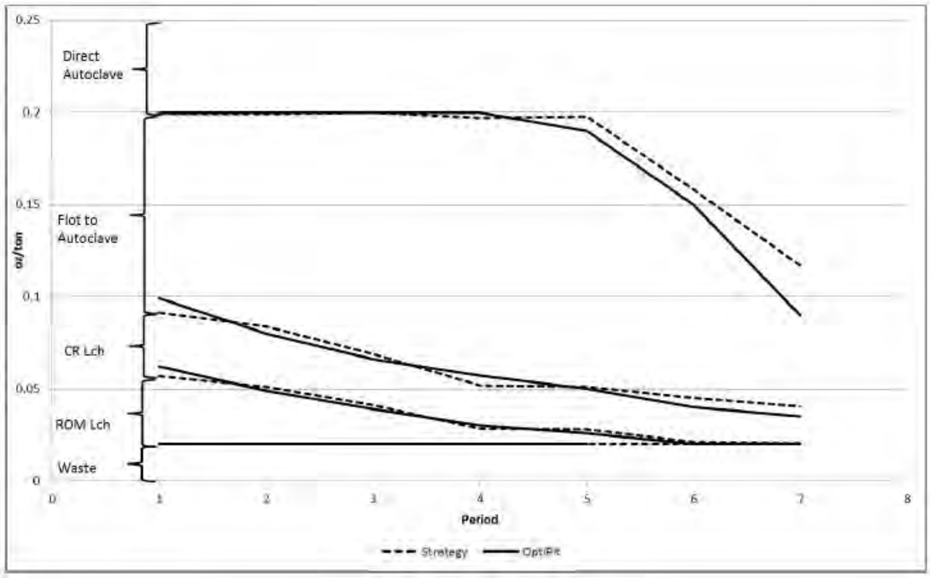

Figure 10 - Comparison of cut-off grade policy between OptiPit® and Strategy.

Finally, let's consider a more realistic scenario where mining is limited to 20Mt per year. Tables 10 and 11 show the yearly schedules and optimised cut-off grades for OptiPit® and Strategy respectively. Figure 11 shows the comparison between cut-off grade policies and as before, the results are virtually identical with less than 0.19% difference in NPV.

Table 10 - OptiPit® optimised schedules and optimum cut-off grades for smoothed tons (after Dagdelen and Kawahata, 2007).

Year	Mining	ROM Lch			CR Lch			Flot to Autoclave			Direct Autoclave			Refining	Profits
	(Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	(koz)	($M)
1	20	0.02	0.029	3.85	0.04	0.052	2.35	0.068	0.13	2.00	0.20	0.279	0.65	373.7	155
2	20	0.02	0.029	3.85	0.04	0.052	2.35	0.068	0.13	2.00	0.20	0.279	0.65	373.7	155
3	20	0.02	0.029	3.85	0.04	0.052	2.35	0.068	0.13	2.00	0.20	0.279	0.65	373.7	155
4	19.98	0.02	0.029	3.39	0.039	0.049	2.35	0.065	0.123	2.00	0.20	0.279	0.65	361.7	149.5
5	14.96	0.02	0.025	1.62	0.03	0.041	2.55	0.057	0.094	1.75	0.20	0.279	0.7	313.5	128.2
6	13.46	0.02	0.023	0.95	0.026	0.037	2.43	0.05	0.084	1.91	0.19	0.274	0.67	294.7	118.8
7	10.28				0.02	0.029	1.95	0.04	0.065	1.94	0.14	0.249	0.66	245.7	95.6
8	7.83				0.02	0.027	1.20	0.035	0.053	1.52	0.087	0.202	0.75	196.9	74.4
Total	125.5			17.5			17.53			15.1			5.4	2533.7	1031.5 NPV @ 15% $614.5

Table 11 – Strategy optimised yearly schedules and optimum cut-off grades for smoothed tons.

Year	Mining	ROM Lch			CR Lch			Flot to Autoclave			Direct Autoclave			Refining	Profits
	(Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	COG (oz/ ton)	Avg grade (oz/ton)	Process-ing (Mtons)	(koz)	($M)
1	20.0	0.02	0.030	3.84	0.041	0.052	2.35	0.069	0.130	1.99	0.20	0.279	0.65	373.97	155.2
2	19.9	0.02	0.030	3.79	0.041	0.052	2.40	0.069	0.130	1.94	0.20	0.279	0.66	372.79	154.8
3	19.9	0.02	0.030	3.79	0.041	0.052	2.36	0.068	0.130	1.99	0.20	0.279	0.65	373.15	154.9
4	17.4	0.02	0.028	2.67	0.036	0.046	2.36	0.061	0.113	1.98	0.20	0.279	0.65	344.45	141.8
5	14.4	0.02	0.025	1.37	0.029	0.040	2.41	0.054	0.091	1.92	0.20	0.278	0.67	308.33	125.3
6	13.1	0.02	0.023	0.77	0.026	0.037	2.55	0.052	0.082	1.76	0.18	0.267	0.70	288.56	116.4
7	11.4	0.02	0.020	0.02	0.021	0.032	2.51	0.045	0.072	1.80	0.15	0.254	0.69	263.66	104.1
8	9.4				0.020	0.028	1.51	0.037	0.060	1.99	0.13	0.242	0.64	230.00	88.5
Total	125.5			16.26			18.44			15.36			5.32	2554.90	1041.1 NPV @ 15% $615.7

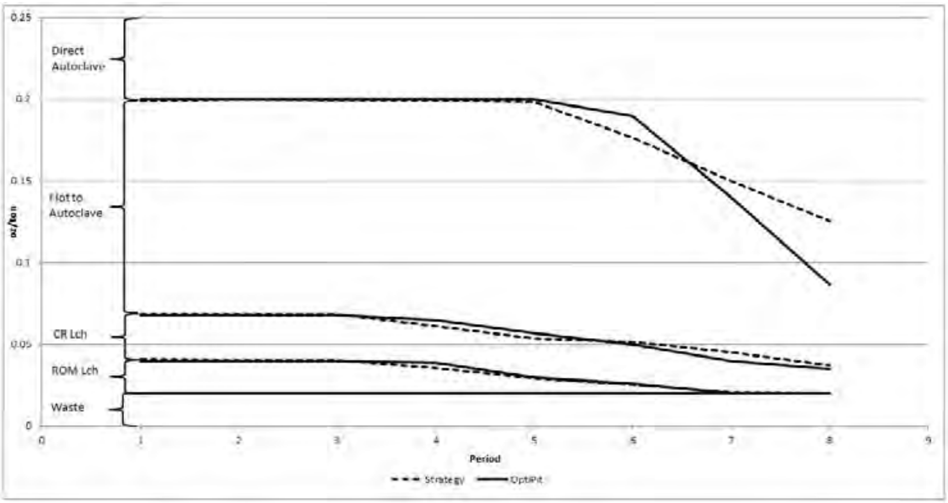

Figure 11 – Comparison of cut-off grade policy between OptiPit® and Strategy.

CONCLUSIONS

We started out by stating that real world problems are complex and that they seldom, if ever, fit the constraints of existing classical optimisation techniques. We are not proponents of any given method, classical or otherwise. Rather we believe that one should not discard any potential technique or class of techniques out of hand, but instead find creative and innovative ways to apply or combine both classical and modern heuristics to tackle complex problems. With this paper we demonstrated that high quality results can be obtained by applying a fast approximate method. Although the time it takes to complete an optimisation varies with complexity of the operation, on average an optimisation takes anything from 3 to 20 minutes. Modern heuristic techniques are quite adaptable and one can easily extend or hybridize them with classical techniques. Evolutionary algorithms are stochastic iterative algorithms which cannot guarantee convergence, but as shown a well-designed algorithm can produce very high quality results quickly. Arguably one of their biggest advantages is that they are agile and easily adaptable (they are based on nature after all) to attack dynamic and complex real world problems. Their agility comes to the fore in that they are particularly well suited to embrace classical optimisation techniques in the quest to solve ever more complex problems.

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