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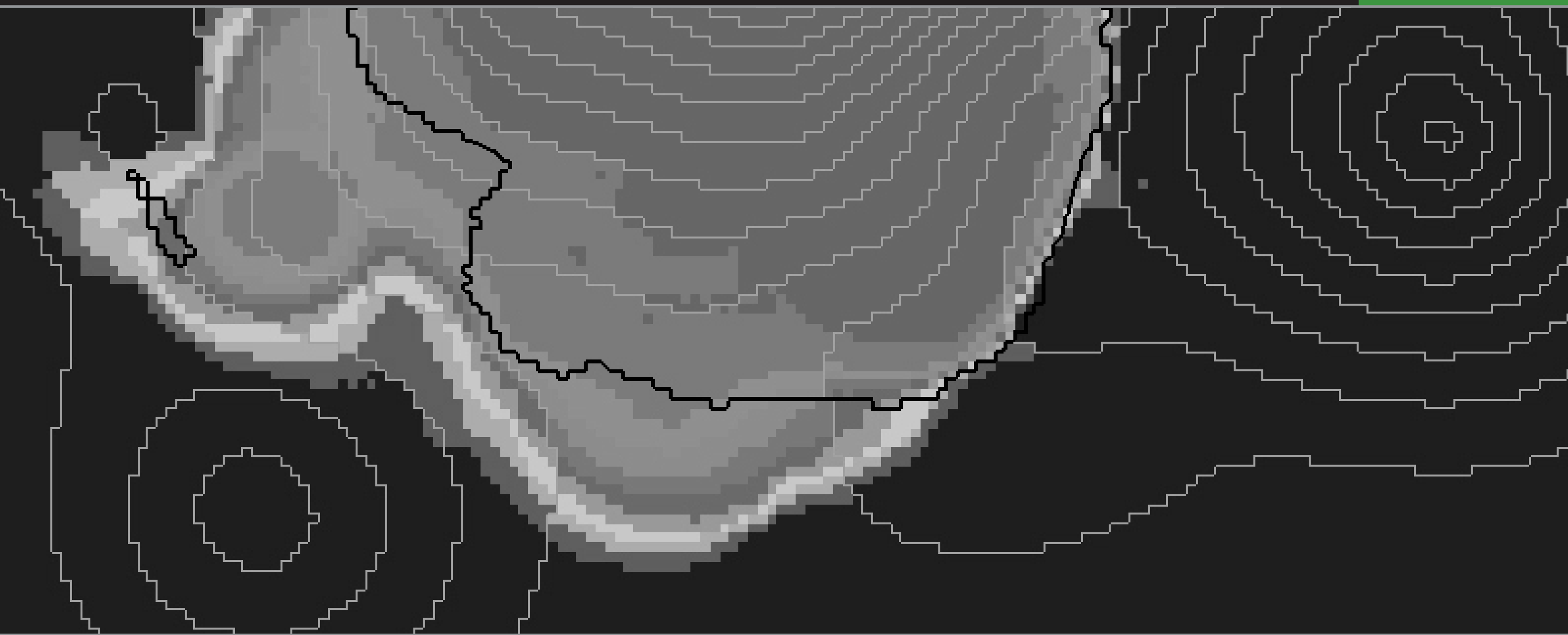

Using Simulation to Quantify Uncertainty in Ultimate Pit Limits and Inform Infrastructure Placement

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ABSTRACT

There is uncertainty in ultimate pit limits due to geologic variation and unpredictable economic landscapes. In this work we show how this uncertainty affects the ultimate pit and how it can be analyzed to improve the mine planning process. A stochastic framework using geostatistical simulation and parametric analysis is used to model the effects of geologic and economic variation on ultimate pit limits and overall project economics. This analysis is made possible by a new, highly efficient pit optimization implementation which can be automated and set up to calculate ultimate pits for hundreds of different scenarios in a matter of hours. Quantifying ultimate pit uncertainty early in the mine planning process allows mining engineers to make informed decisions regarding infrastructure placement, and to mitigate the possibility of incurring substantial costs relocating critical mine facilities.

INTRODUCTION

The mining industry is increasingly concerned with the effects of risk and uncertainty. Uncertain prices, unpredictable global markets, and unknown foreign exchange rates can alter the economic viability of a mining project in substantial ways. This economic uncertainty is compounded by geologic uncertainty. The extent and quality of any given deposit cannot be fully measured and is not known before consequential decisions must be made. These two sources of uncertainty are responsible for the majority of deviation between what happens during operation and what was initially planned. Understanding and explicitly quantifying this uncertainty will lead to better decision making and allow mining engineers and investors to be aware of what may occur.

It is unrealistic to believe that one model, estimated from sparse measurements, is enough to capture both geologic and economic uncertainty and allow for optimal decisions. The entire breadth of uncertainty should be considered, as both upside and downside risks have large impacts on the investment potential and operational efficiency of mining projects. The workflow presented here uses Monte Carlo simulation and parametric analysis together to explicitly analyze the breadth of geologic and economic uncertainty as it applies to ultimate pit calculation and long range mine planning.

In open pit mining, the ultimate pit represents the limit of extraction such that mining any more material would require the removal of so much waste as to make any extra ore irrelevant. The ultimate pit is used to assess the economic viability of the project and to guide the mine planning process. It is generally the first stage in overall site planning as other infrastructure will be placed to avoid intersecting the pit limits and sterilizing ore. The ultimate pit is based on geotechnical, geologic, and economic parameters. All of these parameters are uncertain due to sparse measurements, uncertain markets and other risks. These parameters have complex and non-linear effects on the ultimate pit, which motivates the use of Monte Carlo simulation.

In this paper, we propose a workflow for explicitly analyzing geologic, geotechnical, and economic uncertainty as it affects the ultimate pit in order to better understand the risks associated with any given mining project. The workflow allows for the creation of various figures and maps which summarize the risk and allow for risk-qualified decision making. Ultimate pit uncertainty is also translated into a probability model which is useful in both mine design and project evaluation. This workflow is made possible by an efficient optimal ultimate pit calculator which is able to analyze hundreds of different possibilities in a few hours instead of a few days or weeks. In the remaining sections we perform a brief review of the current literature, explain the workflow, and show a case study. We then discuss the results, compare them with conventional estimation-based techniques, and draw conclusions.

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LITERATURE REVIEW

The ultimate pit represents the final pit contour such that all economic ore is extracted and all unnecessary waste is left in place. This problem has been expressed formally by Hochbaum and Chen (2000) as:

$$\text{Maximize} \quad \sum_{i \in V} b_i \cdot x_i \tag{1}$$

$$\text{subject to} \quad x_j - x_i \geq 0 \qquad \forall (i,j) \in E \tag{2}$$

$$0 \leq x_i \leq 1 \qquad \text{integer, } i \in V \tag{3}$$

Where the block model of the deposit has been re-expressed as a directed graph $G = (V,E)$ where each block is a node in V. Dependencies, dictated by geotechnical constraints, are expressed as edges in the set E. The economic block value b will then be used to determine the integer vector x which indicates if a given block is extracted or left in place.

Equations 1 to 3 have traditionally been solved using the Lerchs and Grossmann algorithm introduced in Lerchs and Grossmann (1965). However, in the same paper Lerchs and Grossmann indicate that the ultimate pit problem could be expressed as a flow problem. Picard (1976) provided the mathematical justification by proving that the selection problem is equivalent to computing the maximum valued closure of a directed graph. As a consequence, sophisticated network flow algorithms can be used in place of the Lerchs and Grossman algorithm and calculate identical results in a fraction of the time. The push-relabel algorithm of Goldberg (1988), and the pseudoflow algorithm of Hochbaum (2001) are two such sophisticated alternatives.

Hochbaum and Chen (2000) investigated the performance of the push-relabel algorithm and the Lerchs and Grossmann algorithm; their study showed that the push-relabel algorithm outperformed the Lerchs and Grossmann algorithm in nearly all cases. When the number of nodes is large, greater than several million, network flow algorithms perform orders of magnitude faster and compute precisely the same results.

Parametric Analysis

Parametric analysis, introduced in the same 1965 paper by Lerchs and Grossmann, is a technique to approximate an optimum mining sequence by calculating several nested pits. This is commonly called the nested Lerchs and Grossmann algorithm. The block values are decreased by some constant and Equations 1 to 3 are solved again. This reduction serves to constrain the volume of the pit and generate a smaller nested pit. When this process is repeated, several nested pits are generated which, when taken as a sequence, extract the highest valued blocks first. However, reducing the block values by a constant does not have an intuitive relationship with the inputs to the block value calculation and, therefore, an alternative reduction strategy is generally employed.

Matheron (1975) introduced a form of parametric analysis where the block value b is expressed in terms of a parameter λ as in Equation 4.

$$b_i = \lambda \cdot c_i + d_i \tag{4}$$

where c_i is the sum of the terms linearly dependent on λ and d_i is the sum of the independent terms. In practice, d is taken to be the costs associated with extracting, processing, transporting, and selling the block, and c is taken to be any revenue. In this case λ is called a revenue factor after Whittle (1989). The ultimate pit is calculated for many revenue factors $\lambda \geq 0$ to generate the nested pits.

Geostatistics and Simulation

Determining the values of the c and d terms in Equation 4 is both site and commodity specific and depends on many different parameters. Many of the parameters are local, in that they vary by location and depend on some geologic attribute such as metal content, rock type, specific gravity, etc. These geologic attributes must be known at every location within the volume of interest to inform the economic block value, however they cannot be directly measured at every location. Therefore, geologists and mining engineers have turned to the field of geostatistics to inform robust interpolation and extrapolation techniques to fill in the gaps. These techniques are based on the sound application of geology and statistics to generate fully sampled models which can then be used in downstream studies. The theory and modern practice of mining geostatistics is discussed in Rossi and Deutsch (2014).

Estimation based techniques such as inverse distance or kriging are only capable of providing one model which is smooth by construction and possibly systematically biased. Geologic uncertainty cannot be captured with a parameter and therefore a single geologic model is not enough. Instead geostatisticians have adopted a stochastic framework based on Monte Carlo simulation. Simulation techniques such as sequential Gaussian simulation, Isaaks (1990), or sequential indicator simulation, Alabert (1987), are free of conditional bias and provide many different equiprobable realizations which, when analyzed together, sample the underlying geologic uncertainty.

Uncertainty in Mine Planning

The results of geostatistical simulation have been used to great effect in mine planning before. Dimitrakopoulus (2002) analyzed the effect of geologic uncertainty on the ultimate pit of a disseminated, low-grade gold deposit and found that the realizations departed substantially from the kriged estimate. In Dimitrakopoulus (2007), the orebody uncertainty is used to determine designs that perform well in the presence of uncertainty. The authors indicate that designs based on stochastic mine planning have led to substantial increases in net present value as the entire range of uncertainty is analyzed.

In Monkhouse (2005), the authors advocate moving beyond naive optimization of a single model of the subsurface and instead urge practitioners to use all sources of uncertainty to make better plans and decisions. Plans that use uncertainty can be developed to achieve optimal results across a reasonable range of real world inputs. We offer a workflow to use that underlying uncertainty and a solution to compute the results quickly and efficiently.

Even with these previous studies there is more to be done to analyze uncertainty in the ultimate pit. One of the drawbacks to sensitivity studies and simulation in general, is the extra computation time and professional time required. We have developed a workflow which can be used to capture this uncertainty and summarize it effectively using fast, robust, and currently commercially available software.

WORKFLOW

Typical results from a long range mine planning exercise for a feasibility study (or during production) include a series of pit shells, a pit by pit graph, and a table of metrics for the chosen ultimate pit. Traditionally, these results are based off of a single estimated model and therefore have no consideration of geologic uncertainty. Stochastic methods will be used to add more information to these results and account for all sources of uncertainty. A conceptual depiction of such a change is shown in Figure 1. The pit shells will be replaced with many different possible pit shells. Uncertainty in the pit by pit graph will be depicted with error bars. The various metrics will be replaced with histograms showing the distribution. Additionally, a further compilation step is introduced to generate a probabilistic model of the pit shells.

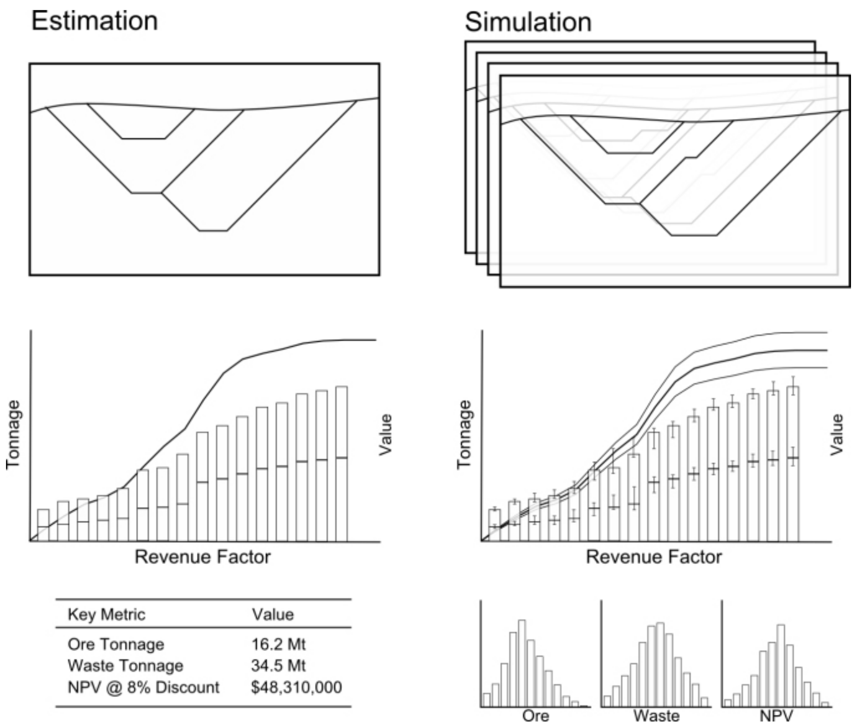

Figure 1. The left side shows the results of standard estimation based long range mine planning; the right side shows how using simulation changes the results to account for uncertainty.

The workflow is shown in Figure 2. This entire workflow can be done manually; however as the number of realizations increases, this becomes practically infeasible. Simple scripts will be used to loop through the realizations and synthesize the results. There are three inputs to the ultimate pit uncertainty workflow: a geostatistical simulation model of the subsurface, distributions of the input economic parameters, and a distribution of the geotechnical parameters.

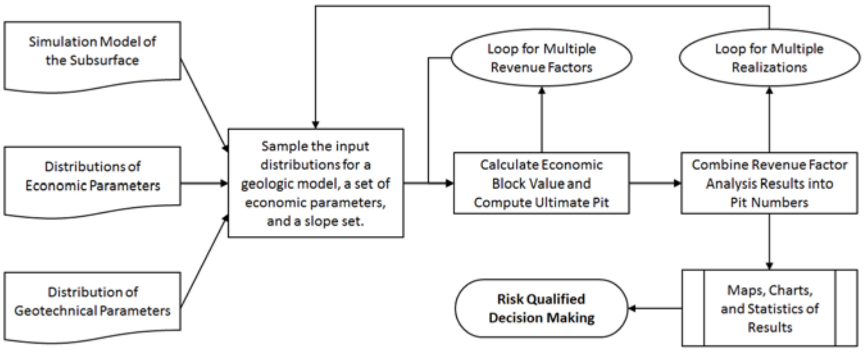

Figure 2. Flowchart showing the proposed workflow for analyzing ultimate pit uncertainty.

To achieve the improvements shown in Figure 1, a simulation model of the subsurface is required to generate equiprobable realizations of the underlying geology. The geostatistical simulation workflow to generate these results will not be discussed in this paper. However, practitioners of simulation should be mindful of recreating the input statistics (distributions, correlations, variograms, etc.) and ensuring the validity of the simulation as a whole. Issues of volume variance and support should be considered, however this may be avoided by simulating at the data scale before averaging to the relevant scale for mine planning.

To analyze uncertainty in the economic parameters, we introduce a stochastic economic block value function. At its core, the economic block value function is simply revenues less costs. A simplistic economic block value formula may have the following form:

$$b_i(\lambda) = \lambda(T_o \cdot g \cdot r \cdot P) - T_o \cdot PC - T \cdot MC \quad (5)$$

Where λ is the revenue factor, T_o is the tonnage of ore, g is the grade, r is the recovery or percentage of product recovered, P is the commodity price, PC is the processing cost, T is the total tonnage and MC is the mining cost. This is a very basic economic block value function and often more complicated functions are used in practice. Normally several different possible processes are defined. Selling costs and different mining costs are included. Multiple factors are applied based on rock type, location, or other parameters. Geometallurgical attributes, contaminants and other local inputs are also often included.

By using geostatistical simulation, we have accounted for uncertainty in the location dependent inputs to the economic block value calculation, such as grade, as they vary between geologic realizations. However, some of the global inputs such as commodity price are also variable and their values are uncertain. To account for uncertainty in these global parameters, we will define some distribution which captures the parameter in question and sample it once for each realization. This leads to a stochastic economic block value function that will have different global economic parameters by realization and therefore account for uncertainty. If production data or some other information is available, this may be done explicitly without resorting to assumptions regarding the character of the underlying distribution.

The geotechnical information describes the allowable pit slopes for every block and can take many forms. Commonly, slope requirements are defined by azimuth within different zones. Geotechnical uncertainty can be included by having different slope definitions for each realization or by basing the slopes on zones which have been simulated using some form of categorical simulation. Depending on the geotechnical context of the area in question, it may make sense to hold the slope definitions constant.

The remainder of the workflow is to draw from the input distributions, fully sampling the space of uncertainty, and then perform parametric analysis with that particular geologic model, economic block value function, and slope definition. The sampling and parametric analysis is then completed for many realizations. After a reasonable number of realizations, on the order of a few hundred, have been completed, the results are synthesized.

To aid in describing how to synthesize the results, we will use the following notation for the pits. Recall that each pit vector, calculated from equations 1 to 3, is an integer array with a 1 for each block that lies within the ultimate pit limits and 0 for any block outside. Denote a single pit vector $x_{l,\lambda}$ where l is the realization index and λ is the revenue factor. Let L and Λ represent the set of realizations and revenue factors respectively. One common summary is the pit number. The pit number is calculated as follows:

$$PN_i(l) = |\Lambda| - \sum_{\lambda \in \Lambda} x_{l,\lambda,i} + 1 \quad i \in V \qquad (6)$$

The pit number is set to 0 for air blocks, and to a large number for blocks which are outside the largest pit.

The pit numbers correspond directly to the pit by pit graph. Ore and waste tonnages may be calculated simply, and with many, realizations error bars may be added. The error bars indicate the variability in both ore and waste for that particular revenue factor. Histograms of key indicators for any given revenue factor may be extracted and reported. The discounted cash curve and net present value depend on determining an extraction sequence which honors production and extraction constraints. Determining an extraction sequence is beyond the scope of this paper.

A further useful summary of ultimate pit uncertainty is the probability model. The probability model is similar to the hybrid pits of Whittle and Bozorgebrahimi (2004). It is defined:

$$PM_i(\lambda) = \frac{1}{|L|} \times \sum_{l \in L} x_{l,\lambda,i} \quad i \in V \tag{7}$$

The probability model indicates what the probability is for a given block to be extracted for a given revenue factor. For example, if it is assumed that the ultimate pit occurs at some revenue factor λ_U, $PM(\lambda_U)$ can form the basis for designing the ultimate pit and the probability models for $\lambda < \lambda_U$ can be used to assist in sequencing the mining process to extract high probability ore first. The intersection of $PM(\lambda_U)$ and the topography can also be plotted on a map which indicates the range of possible locations of the final pit crest.

CASE STUDY

A case study of a small copper deposit is carried out to test the workflow and analyze ultimate pit uncertainty with real data. The deposit is modeled using both estimation and simulation techniques. Parametric analysis is completed using stochastic economic block value functions and varying slopes. The results are then synthesized and ultimate pit uncertainty is assessed to inform mine valuation and mine planning.

Geologic Modeling

There are 43 drillholes with a combined length of 1450 meters in the area of interest. There are approximately 400 assays measuring copper content and rock type. There are five rock types associated with the host rock, sedimentary layers, quartz, andesite and the high grade copper bearing dyke. This is an exploration dataset and the deposit is sparsely sampled so there is substantial geologic variation. This dataset is simplistic, with only one product; however the workflow is suitably general for more realistic cases.

An implicit model of the rock types using signed distance functions was generated to inform the extent and character of the domains. This model was conditioned to drillhole data and existing geologic interpretation. Within each domain, experimental variograms of copper grade were calculated and modeled. These variogram models were then used to generate a best estimate model using ordinary kriging.

The copper assays were then transformed to facilitate sequential Gaussian simulation. Normal score variograms were calculated and modeled, and then copper grade within the domains was simulated. Several of the realizations were checked visually and many more were assessed for variogram and histogram reproduction. Two of the simulated realizations are shown in Figure 3.

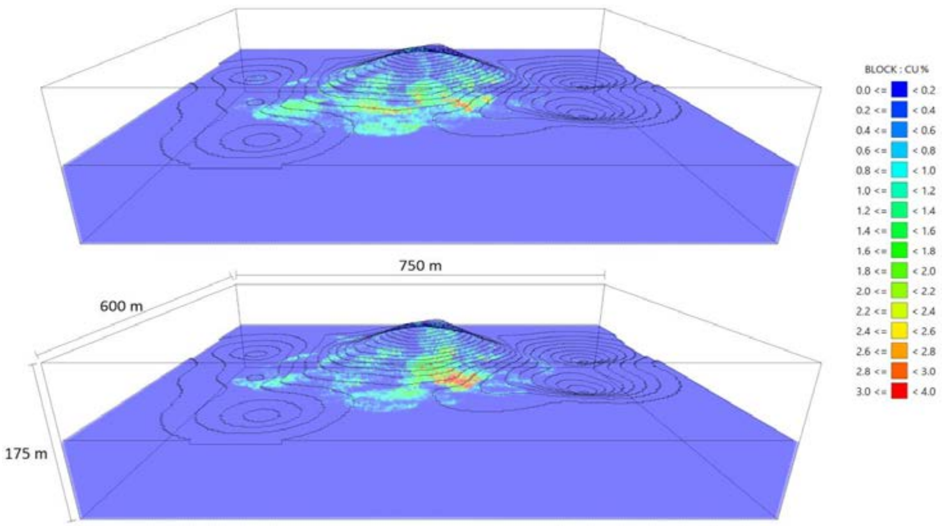

Figure 3. Two realizations of copper grade. Blocks are displayed semi-transparent and colored by copper content.

Ultimate Pit Calculation

For the case study, the basic economic block value function defined in Equation 5 is used. Lacking production data and any other insight, the various global parameters are assumed to be normally distributed with the means and standard deviations given in Table 1. These assumptions are made without loss of generality; if more detailed information existed, it could easily be implemented into the scripted workflow.

Table 1. Global parameters varied in the case study.

Parameter	Mean	Standard Deviation
Mining Cost	2.0 $/t	0.2 $/t
Recovery	75 %	1 %
Price	2.2 $/lb	0.2 $/lb
Processing Cost	4.8 $/t	0.1 $/t
Overall Pit Slope	45 °	1 °

Forty-six revenue factors uniformly distributed between 0.3 and 1.2 are used in the parametric analysis. These nested pits form the basis for the pit by pit graph and the sequence used to generate the discounted cash flows. A sequence is calculated using an assumed mining rate of 0.5 Mt per year and a bench lag of three benches.

A single realization consists of one geologic model generated using sequential Gaussian simulation, and a set of parameters sampled from Table 1. Five hundred realizations were calculated. The entire process required the calculation of 23,000 ultimate pits on a model with just over 1.7 million blocks. The ultimate pit uncertainty simulation, including the geologic simulation, finished overnight.

RESULTS AND DISCUSSION

The pit by pit graph and distributions of several key performance indicators are included in Figure 4. There is substantial variation in the tonnages and indicators across all revenue factors. The ore and waste tonnage bars indicate the mean value across all of the realizations and the error bars indicate the 10th and 90th percentile. The discounted cumulative cash flow is shown as three lines with the bold line indicating the mean and the two surrounding lines as the 10th and 90th percentile. The conventional estimation based results using the kriged model and average values for all other input parameters is shown as the dashed black line.

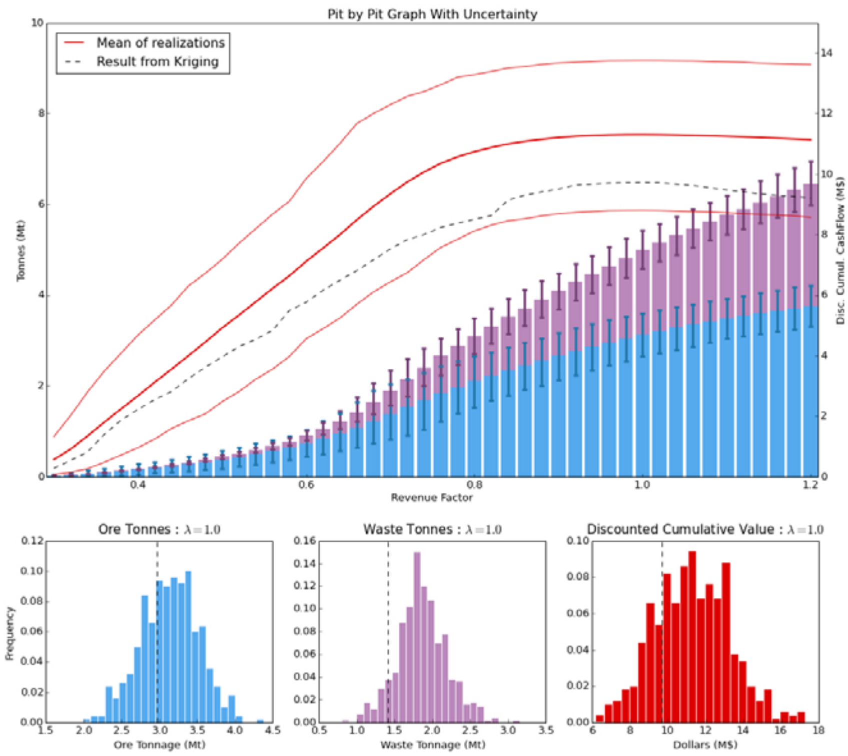

Figure 4. Results of the case study for ultimate pit uncertainty. Error bars and lines indicate the 10th and 90th percentile. The histograms are for revenue factor of 1.0. The dashed line shows the result from a conventional estimation based workflow.

The variation in the key performance indicators emphasizes the need to consider uncertainty in parametric analysis and ultimate pit calculation. Strategic decisions are based on the value and extent of the ultimate pit and if those values are demonstrably variable, those decisions should adapt. It is one thing for a mining engineer to decide on the fleet to purchase based on a single number, but if the distribution is known, the fleet can be purchased with the appropriate amount of flexibility in mind. Also, in this case study, the average kriging model with average economic parameters does not give an average assessment after the ultimate pit and long range planning analysis is completed. In this small case study, the expected discounted value across all realizations is 11.3M$ with a standard deviation of 2M$, the kriged model indicated a value of 9.7M$. Deviation is expected, as the ultimate pit calculation is not linear with respect to the input parameters and average inputs do not guarantee average outputs; however, it is not guaranteed that the average assessment be lower than the average.

A probability model was extracted for revenue factor 1; the intersection of this model with the topography is shown in Figure 5. Every block is colored based on its likelihood to be within the pit. The red innermost blocks are in all 500 pits, the blue outside blocks are in none of the pits. The pit crest of the single model from kriging is also shown as a dashed black line. This model indicates where the crest could be, based on the underlying uncertainty. From a mine planning perspective, a continuum of results is much more valuable. In this case study the pit wall is much less variable along the east side, however there are a great many pits which extend towards the west. Infrastructure can now be placed appropriately accounting for where the pit may be in the future.

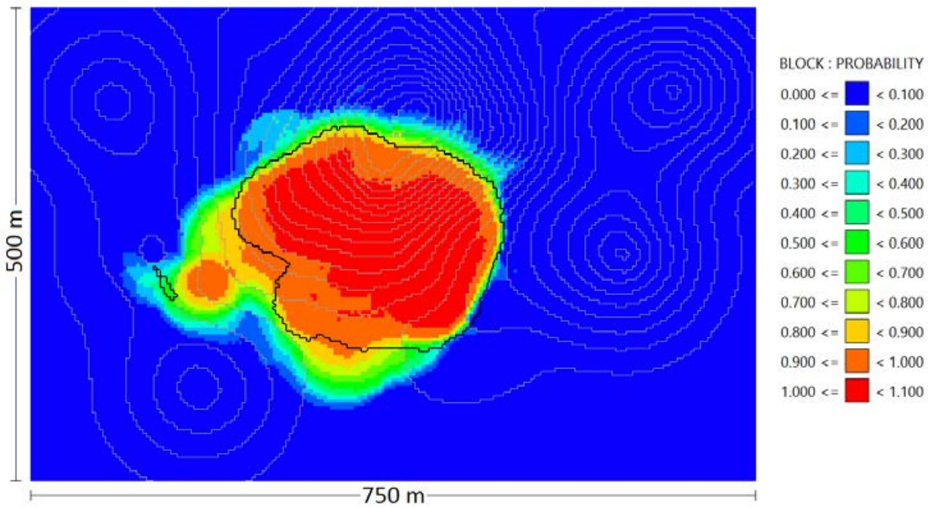

Figure 5. Intersection of the probability model for revenue factor = 1 with the surface. The ultimate pit crest for the estimated model is shown as the thick black line.

CONCLUSIONS

In this paper we have proposed a workflow to capture ultimate pit uncertainty using Monte Carlo simulation and an efficient ultimate pit solver. We have generated results which sample the entire space of uncertainty and allow for risk qualified decision making. Variability in the subsurface and all other input parameters is explicitly translated through the long range mine planning transfer function to analyze uncertainty in the ultimate pit.

A case study was completed on a small exploration dataset with 43 drillholes. Kriging and simulation was used to build geologic models. The geologic models were translated into economic block values using an average function for the kriged model, and a stochastic function for each realization sampling the underlying economic parameters. Parametric analysis was then completed using a range of revenue factors for all realizations. The results were then synthesized into figures and graphs which summarize the risk inherent in the mining project.

The results of the case study emphasize the need for explicitly analyzing uncertainty. The mean Monte Carlo result shows a larger pit which generates more revenue and requires more ore and waste to be mined than the conventional analysis based on the mean input parameters would suggest. This discrepancy may lead to suboptimal decisions and plans which do not consider the underlying uncertainty.

Analyzing uncertainty at an early stage allows for plans to be developed which account for what could occur. The simulation based workflow presented here is completed using commercially available software with a reasonable amount of professional time expended. Uncertainties in geologic, economic, and geotechnical parameters can be quantified and analyzed which allows for flexible plans to be developed and appropriate risk qualified decisions to be made.

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